

THE TIP CLEARANCE EFFECTS ON THE CENTRIFUGAL COMPRESSOR VANELESS DIFFUSER FLOW FIELDS AT OFF-DESIGN CONDITIONS

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ABSTRACT

Three centrifugal compressor vaneless diffuser configurations, varying in diffuser width, were studied experimentally. Each diffuser configuration was tested with four different tip clearances. The stage overall performance along with the diffuser flow fields were studied. The diffuser flow fields were studied with probe traverses for total temperature, total pressure, and flow angle, along with static pressure measurements. The diffuser flow fields were studied at the diffuser inlet and outlet at the compressor design rotational speed near the stall ($1.04 \cdot q_{m, \text{stall}}$) and choke ($0.96 \cdot q_{m, \text{choke}}$) conditions. The increasing tip clearance is seen as a larger secondary flow region present at the diffuser inlet near the shroud. This causes additional losses which are seen as lower compressor stage efficiency. The effect is more profound at higher mass flows. The flow velocity distributions at different circumferential locations showed significant differences at off-design conditions. One particular circumferential location showed higher or lower velocities depending on whether the mass flow was higher or lower than the design flow, and the phenomenon is stronger at the low flow.

NOMENCLATURE

b	channel width/blade height	m	Ω	angular velocity	rad/s
c	flow velocity	m/s	π	pressure ratio	-
c_p	specific heat capacity	J/kgK	Subscripts		
h	specific enthalpy	J/kg	1	compressor inlet	
N_s	specific speed	-	2	impeller outlet/diffuser inlet	
q_m	mass flow	kg/s	2'	beginning of the pinch	
q_v	volume flow	m ³ /s	5	compressor outlet	
R	specific gas constant	J/kgK	des	design condition	
T	temperature	K	ref	reference condition	
t	tip clearance	m	s	isentropic	
η	efficiency	-	tt	total-to-total	

INTRODUCTION

Centrifugal compressors are widely used e.g. in internal combustion engine turbochargers, oil and gas industry, refrigeration processes, waste water treatment, and small gas turbines. The increasing electricity prices and the constant demand for more energy efficient and environmentally friendlier processes sets new challenges for design engineers. Even though centrifugal compressors have been successfully designed, manufactured and operated for decades, there is still a clear need for further research to enable the design engineers to design better compressors faster and more accurately.

As centrifugal compressor impellers have been studied more than the stationary parts, there is substantial potential for improving the compressor performance in the stationary parts (Kim et al. (2002); Issac et al. (2003)).

Pinch, meaning the reduction of the diffuser width after the impeller, is quite commonly used design practice even though its effects on the compressor performance are not fully understood. Previous study by the current authors (Jaatinen et al. (2011)) showed that moderate width reductions improve the compressor stage performance. The improvement was seen as a stage efficiency increase, and in addition to the efficiency, also the stage pressure ratio increased with the pinch. The benefits of a moderate pinch was also observed by Di Liberti et al. (1996).

On the other hand, literature surveys indicate that excessive pinches deteriorate the stage efficiency (Ludtke (1983); Ferrara et al. (2002a,b); Cellai et al. (2003)).

In centrifugal compressor vaneless diffusers, the blade wakes mix out rapidly with the high momentum jet whilst the passage wakes mix slowly. In addition, the circumferential variations mix out rapidly whilst the axial variations tend to persist. At the higher flow rates than the design mass flow, the stronger secondary flows prevent the rapid mixing of the circumferential variations (Pinarbasi and Johnson (1994, 1995, 1996)). The impeller outflow has four regions which are potential sources of loss. The regions are the blade wake, the shear layer between the passage wake and jet, the thickened hub boundary layer, and the interaction region between the secondary flow within the blade wake with the passage vortex (Pinarbasi (2009)).

The passage wake is highly affected by the tip clearance. With a reduced clearance, there was no flow reversal at the shroud and the flow was less tangential in the wake. With larger clearances, the wake was larger and located closer to the hub (Schleer and Abhari (2008)).

Obviously, the tip clearance also affects the machine performance. It has been universally long known that increasing the tip clearance decreases the stage efficiency and pressure ratio (Pampreen (1973); Eisenlohr and Chladek (1994); Palmer and Waterman (1995)), and the efficiency drop due to the increased tip clearance is higher at the higher mass flows (Mashimo et al. (1979); Senoo and Ishida (1987)).

For axial flow machines, both the tip clearance jet and the tip clearance vortex produce significant mean velocity gradients that contribute to the production of vorticity and turbulent kinetic energy, and these velocity gradients are the main loss source (You et al. (2007)). It is plausible to assume that the root causes for losses caused by the tip clearance flow are similar for both the axial and radial flow machines.

Centrifugal compressors can be assumed to be closely adiabatic so only the entropy creation by irreversibilities have a major contribution to the loss of efficiency. In the turbulent boundary layer, most of the entropy generation occurs within the laminar sublayer and the logarithmic region. In the diffuser, the sources of losses are the mixing of blade and passage wakes, and the losses increase with the swirl angle at the impeller exit (Denton (1993)).

In this paper, the vaneless diffuser flow fields at off-design conditions are studied. Three different diffuser widths are examined with four different tip clearances each. The flow fields are studied at the design rotational speed at operating conditions close to choke and stall. Preliminary results for one of the diffuser design have been reported in Jaatinen et al. (2012b).

The paper is comprised of four main parts. In the first part, the study is put into a wider perspective by means of a literature survey; in the second part, the measurement setup and the studied designs are explained; in the third part, the most important results are presented and discussed; and finally, in the fourth part, conclusions are drawn and the results are discussed.

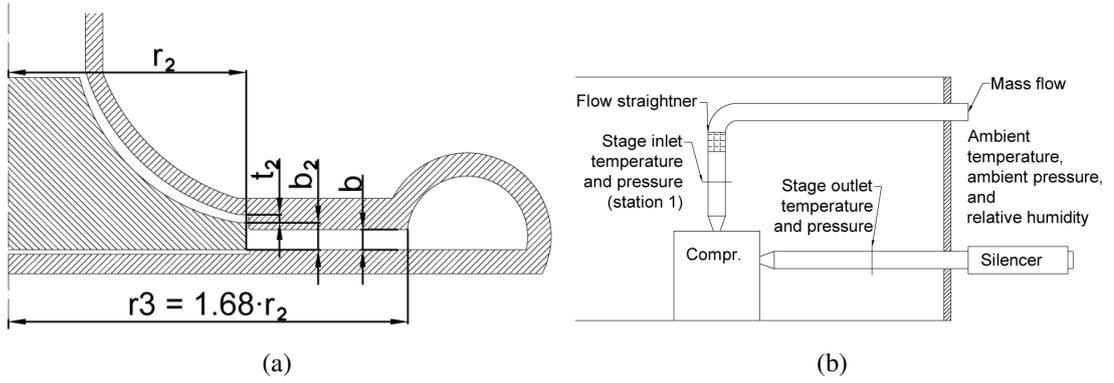


Figure 1: Schematic view of the pinch and the layout of the test stand

Table 1: Studied Designs

Case 1 (unpinched)			Case 2 (10% pinch)			Case 3 (15% pinch)		
t_2/b_2	$b/(b_2 + t_2)$	r_3/r_2	t_2/b_2	$b/(b_2 + t_2)$	r_3/r_2	t_2/b_2	$b/(b_2 + t_2)$	r_3/r_2
0.027	1.00	1.68	0.027	0.906	1.68	0.027	0.854	1.68
0.053	1.00	1.68	0.053	0.908	1.68	0.053	0.857	1.68
0.082	1.00	1.68	0.082	0.911	1.68	0.082	0.860	1.68
0.106	1.00	1.68	0.106	0.913	1.68	0.106	0.863	1.68

STUDIED DESIGNS AND TEST COMPRESSOR

Studied Designs

Three different vaneless diffusers were studied: the original, unpinched one, and two pinched diffusers. All three designs were parallel wall vaneless diffusers, and the pinch was implemented only to the shroud wall. The tested diffuser widths $b/(b_2 + t_2)$ were 1.0, 0.903, and 0.854. The pinch shape was a straight wall, normal to the shroud, and a rounding of a quarter of a circle. The pinch began at a radius ratio r_2'/r_2 of 1.01. A schematic view of the pinch is shown in Fig. 1.

All three diffusers were tested with four different tip clearances. The relative clearances t_2/b_2 were 0.027, 0.053, 0.082, and 0.106. The smallest clearance was the original one, and the tip clearance was increased by shimming the shroud side casing. The shimming leads to slightly increased diffuser width. In this study, the change in diffuser width with the smallest and largest clearance was less than one percent. In the above-mentioned previous study (Jaatinen et al. (2011)), it was observed that a change of this small did not significantly affect the stage performance.

A summary of the studied designs, the tip clearances, and the diffuser widths is presented in Table 1.

Test Compressor

The test compressor is an industrial, single-stage high-speed centrifugal compressor. The compressor is directly driven with a variable-speed drive, and it is equipped with magnetic bearings. In the test stand, the compressor is operated as it would be under normal operation.

The test compressor has an axial inlet and it is equipped with a volute, designed to provide a constant circumferential pressure distribution at the design operating condition. An exit cone is placed after the volute to connect the compressor stage to the piping.

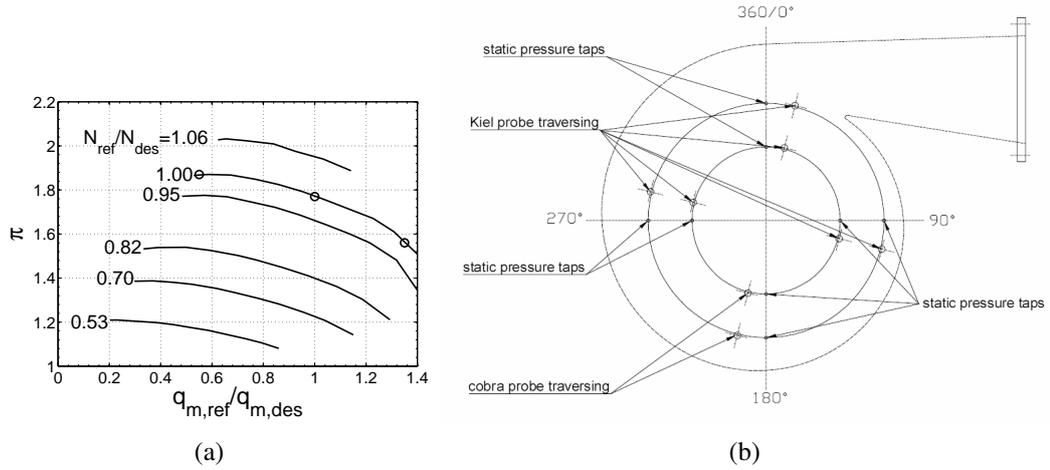


Figure 2: a) Non-dimensionalised compressor map showing the measured points b) Probe traverse and static pressure tap locations

The impeller has seven full and seven splitter blades with a 40° back sweep from the radial direction. The ratio of impeller outlet height to the outlet radius b_2/r_2 is 0.1214.

The compressor design pressure ratio π_{tt} is 1.78, and the design condition Mach number and Reynolds numbers, as defined in ISO 5389 (1992), are 0.92 and $4.4 \cdot 10^5$, respectively. The specific rotational speed is 0.8, and it is defined as

$$N_s = \frac{\Omega \sqrt{q_{v,1}}}{\Delta h_s^{0.75}} \quad (1)$$

MEASUREMENT SETUP

The test stand used in the study intakes from and discharges into the atmosphere. A layout of the test stand is shown in Fig. 1. The incoming mass flow is measured with an ISA 1932 nozzle made according to DIN 1952 (1971). The instrumentation and performance calculation was carried out according to standards: ASME PTC 10 (1965), ISO 5389 (1992), VDI 2045 Part 1 (1993), and VDI 2045 Part 2 (1993). The same test stand was used in Jaatinen et al. (2011). The stage performance is defined with the inlet and discharge measurements of the test stand.

The flow fields were measured at the design rotational speed at mass flows $q_{m,ref}/q_{m,des}$ of 1.35 and 0.55. A non-dimensionalised operating map showing the measured operating points is presented in Fig. 2. In the flow field measurements, the total pressure and flow angles were measured with a cobra probe at the diffuser inlet and outlet at one circumferential location. Total pressure and total temperatures were measured with Kiel probes at three other circumferential locations. The height of the cobra probe is 0.074 times the diffuser width, and the height of the Kiel -probes is 0.196 times the diffuser width. The blockage caused by the probes is minimal since the flow area is large when compared to the probe head areas. Static pressures were measured adjacent to the probe traversing locations. The measurements were performed at the radius ratios r/r_2 of 1.054 and 1.67 at the diffuser inlet and outlet, respectively. The probe traversing locations along with the location of the static pressure measurement taps are shown in Fig. 2.

The relative measurement uncertainties with 94.5% confidence interval for mass flow, efficiency, pressure ratio, and flow velocity are $\pm 0.17\%$, $\pm 0.47\%$, $\pm 0.33\%$, and $\pm 3.8\%$, and the absolute error for flow angle is $\pm 1^\circ$.

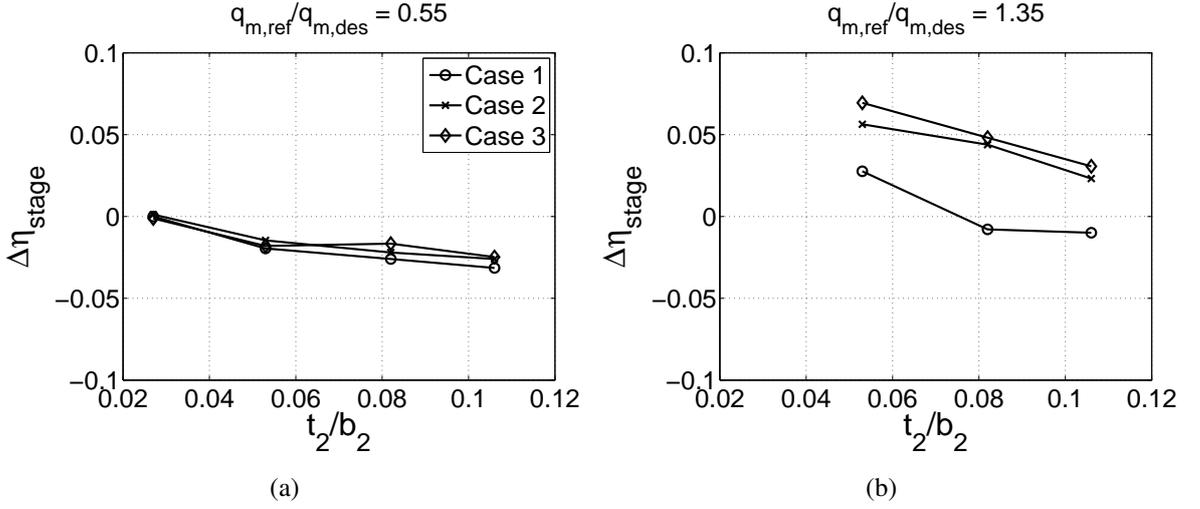


Figure 3: **The stage efficiency drop as a function of the tip clearance. The efficiency is compared to the efficiency of the unpinched diffuser (case 1) at the smallest tip clearance. (a) $q_{m,ref}/q_{des} = 0.55$, and (b) $q_{m,ref}/q_{des} = 1.35$**

When the results were post-processed, it was noted that due to a human error, for the smallest tip clearance, all the flow fields at the high mass flow were measured with a higher mass flow than the other three clearances, and hence are not presented here.

RESULTS

Stage Performance

First the changes in the stage total-to-total efficiencies are studied. This is done to see how the changes in the stage efficiency later compare to changes in the diffuser flow fields. The stage total-to-total efficiency is defined as

$$\eta_{tt,1-5} = \frac{T_{t1} \left(\pi_{tt}^{R/c_p} - 1 \right)}{T_{t5} - T_{t1}} \quad (2)$$

The stage efficiency drop due to the increasing tip clearance is presented in Fig. 3. The efficiency drop is calculated by subtracting the efficiency of the original design (Case 1, the smallest clearance) from each efficiency.

From Fig. 3 it can be seen that the stage efficiency decreases as the tip clearance is increased. At the lower mass flow, the efficiency decrease due to the increase in the tip clearance is smaller than at the higher mass flow, as one would expect. In addition, it can be seen that the pinch improves the stage efficiency, again as one would expect. The narrowest of the three tested diffusers is the best, except at the lowest tip clearance at the high flow where the said diffuser is choked whilst the two other are not. Also, the effect of the diffuser width is higher at the high flow than at the low flow.

Flow Velocity

The absolute flow velocity is calculated with the total pressures, the total temperature measured with the Kiel probes, and the static pressures measured with an adjacent tap. The static pressure is assumed to be constant over the diffuser width.

Velocity distributions at different circumferential locations

The flow velocities at different circumferential locations are shown in Fig. 4. As the results are similar in all three cases, only the Case 3 is shown as an example, and the flow velocities at the diffuser inlet are shown at all three mass flows, and the flow velocities at the diffuser outlet are shown only at the design flow. From Fig. 4 it can be seen that firstly at the diffuser inlet, the slow, secondary flow region is larger with the higher tip clearance. The largest difference in different velocity distributions is always near the shroud. In general, the slowest velocities are seen at 194° which is the farthest measurement location from the volute tongue. In general, the velocity distributions at the diffuser inlet are not that different to each other, and the velocity distributions at the diffuser outlet at the design flow are not that different at different circumferential locations.

It is known that if the compressor is equipped with a volute and if circumferential static pressure distribution, provided by the volute, is even at the design flow, then at the low flow, the circumferential pressure distribution has a positive gradient to the direction of rotation and a negative gradient at the high flow, especially at the diffuser outlet (Shaaban and Seume (2007); Jaatinen et al. (2012a)). The flow velocities at the impeller exit at off-design conditions are shown in Fig. 5. At the low flow, the circumferential location which had the lowest static pressure (Jaatinen et al. (2012a)) has the highest velocity. At the high flow, the same location had the highest static pressure and now it has the slowest velocity.

In the previous study (Jaatinen et al. (2012a)), the minimum or maximum value of the static pressure was always at the circumferential location of 90° which is the next static pressure tap from the volute tongue towards the direction of rotation, and the maximum/minimum velocity was at the same circumferential location. Then, to the direction of rotation, the static pressure increased at the low flow and decreased at the high flow. Similar trend is not seen in the velocity distributions, indicating that other factors contribute to the velocity distributions more than the pressure.

Shaaban and Seume (2007) explained the direction of the pressure gradient. They concluded that when the mass flow is higher than the design mass flow, the flow accelerates in the volute, and this acceleration causes a negative pressure gradient inside the volute at high flow rates, and the negative pressure gradient in the volute is seen as a negative circumferential pressure gradient in the diffuser. At mass flows lower than the design flow, the flow decelerates in the volute, resulting in a positive pressure gradient seen in the diffuser as a positive pressure gradient to the direction of the rotation.

The effect of the tip clearance

In Figs. 4 and 5, the velocity distributions with the largest and the second smallest clearance are shown. At all three tested flows, the effect of the increasing tip clearance is most clearly seen at the diffuser inlet near the shroud. There it can be observed that increasing the tip clearance increases the secondary flow region resulting in larger losses. The effect is more pronounced at the high flow, explaining the larger efficiency decrease at the high flow. At the diffuser outlet, the difference in the velocity distribution is smaller, indicating that the mixing of slow flow region is stronger when the tip clearance is larger. The stronger mixing causes higher losses, partly explaining the loss of efficiency with the increasing tip clearance.

At the higher mass flow, where the effect of tip clearance is larger than at the low or design flows, the effect of tip clearance is smaller when the diffuser width is reduced, or the pinch is increased. This indicates that pinched diffusers are less sensitive to the tip clearance effects. Previously Backman et al. (2007) concluded that compressors with vaneless pinched diffusers are less sensitive to the tip clearance changes. As the pinched diffuser is less sensitive to the tip clearance, it is likely that the pinch counters some losses caused by the tip clearance. Part of the losses caused by the tip clearance flow are seen as an increase in the secondary flow region, which the pinch suppresses, leading to an improved compressor efficiency.

Summary of results

As expected, the stage efficiency decreases as the tip clearance increases, and the effect was more profound at the higher mass flow. The increasing tip clearance increased the secondary flow region present at the diffuser inlet near the shroud which the pinch suppresses. The effect of the increasing tip clearance is not as straightforward at the off-design conditions as it is at the design condition. The velocity distributions at the different circumferential locations showed significant variations especially at the diffuser outlet at the off-design conditions. At the low flow, one circumferential location showed significantly higher velocities and at the high flow, the same location showed the slowest velocities. This followed the trends seen in circumferential static pressure distributions. However, the velocity distributions at other circumferential locations did not follow the pressure trend. The differences in the velocity distributions with respect to the circumferential location were significantly higher at the low flow than they were at the high flow. As the effect of tip clearance on the stage efficiency is higher at the high flow, the other parameters affecting the flow fields are stronger at off-design conditions.

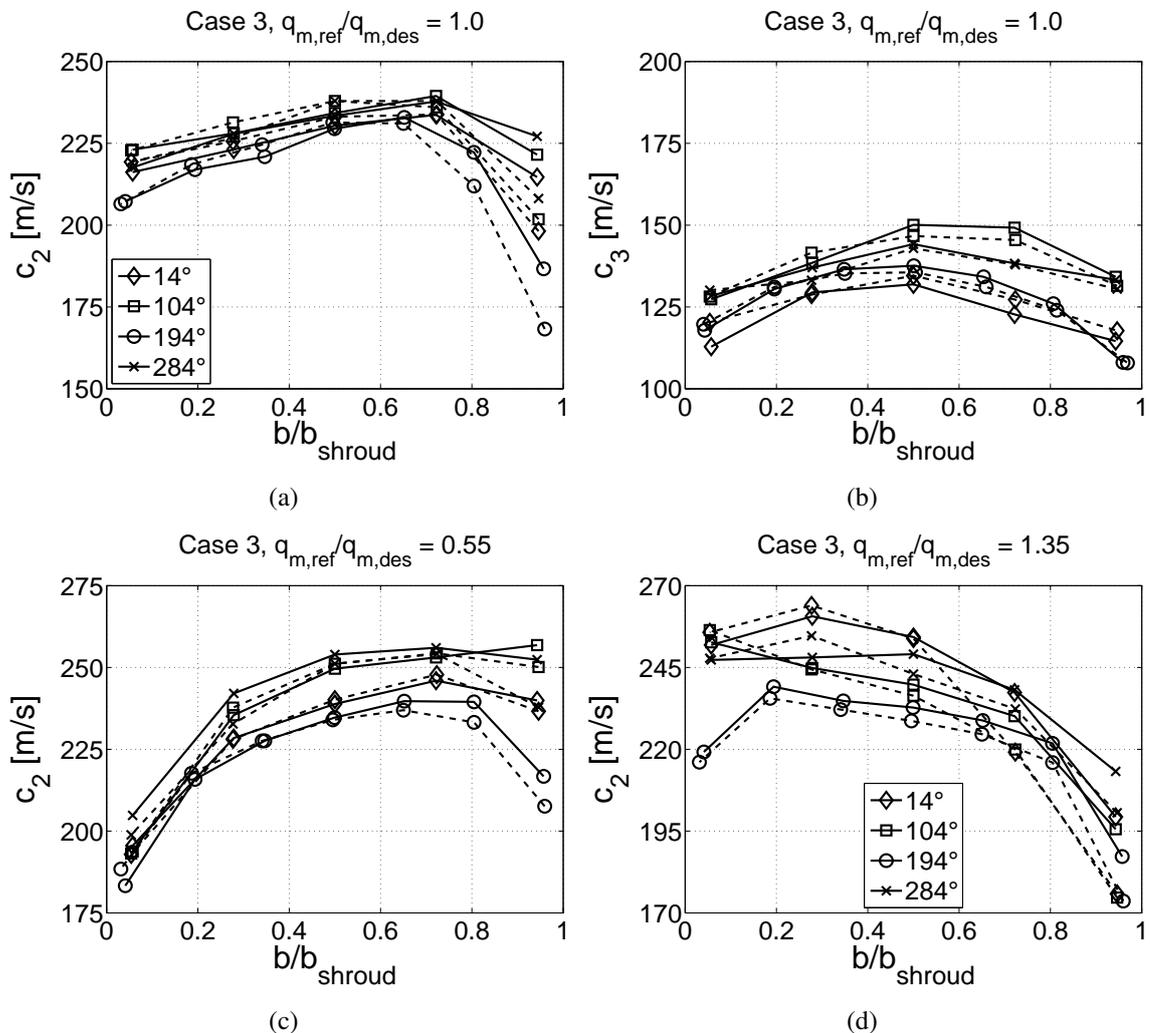


Figure 4: Absolute flow velocities of Case 3 at different circumferential locations, solid line the second smallest clearance and dashed line the largest clearance. (a) diffuser inlet design flow, (b) diffuser outlet design flow, (c) diffuser inlet low flow, and (d) diffuser inlet high flow.

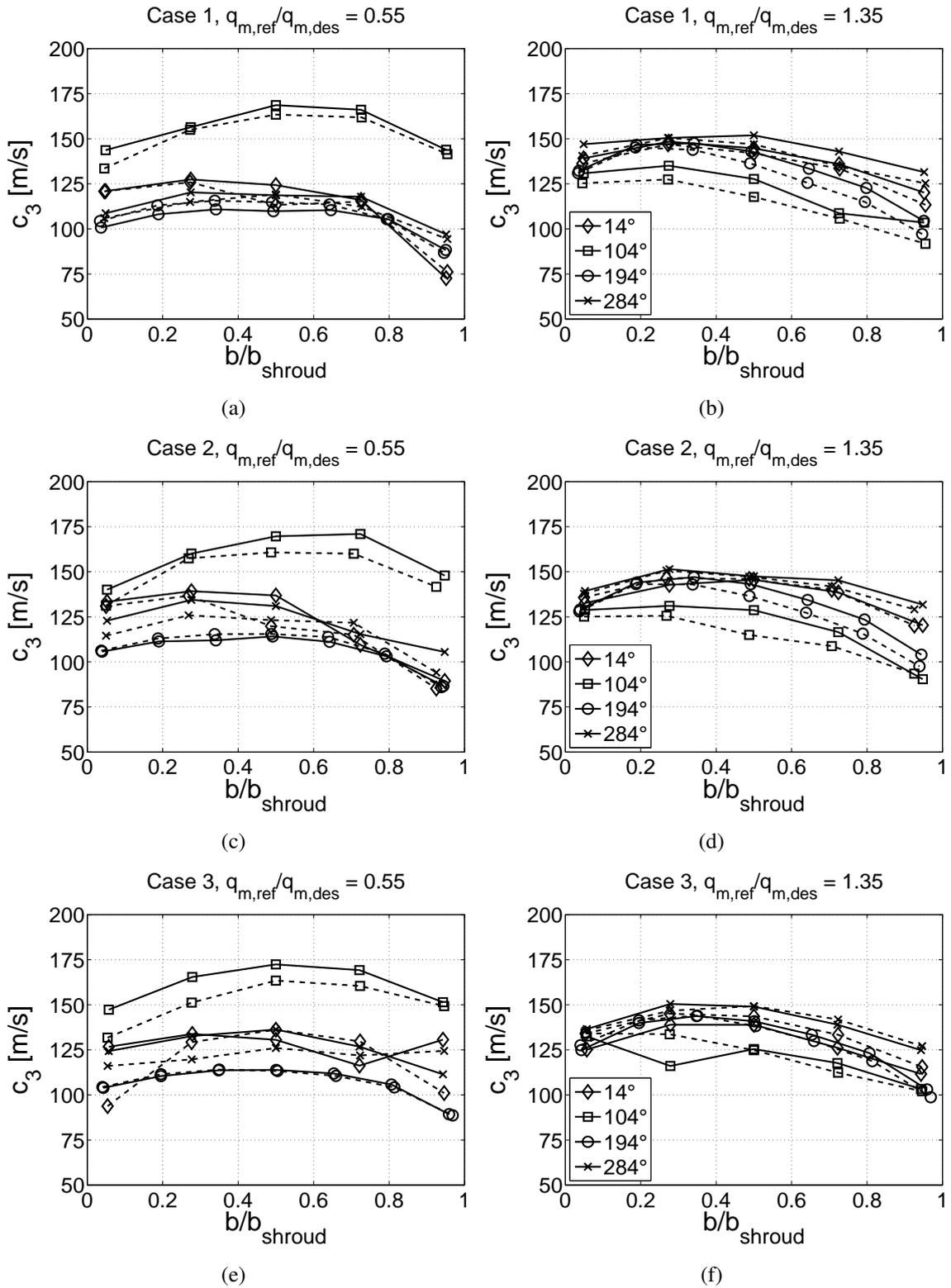


Figure 5: Flow velocities at the diffuser outlet at different circumferential locations, solid line the second smallest clearance and dashed line the largest clearance. Left low flow and right high flow, top Case 1, middle Case 2 and bottom Case 3

CONCLUSIONS

Centrifugal compressors vaneless diffuser flow fields at off-design conditions were studied experimentally. Three different diffuser widths ($b/(b_2 + t_2) = 1.00, 0.903, \text{ and } 0.854$) were studied, and four different tip clearances ($t_2/b_2 = 0.027, 0.053, 0.082, \text{ and } 0.106$) were used for each diffuser. Stage performance was measured according to the relevant standards, and diffuser flow fields were measured using probe traverses for flow angle, total pressure, and total temperature and static pressure measurements. This enabled the axial flow angle and velocity distributions to be examined. The flow fields were studied at the design rotational speed close to the choke ($q_{m,\text{ref}}/q_{m,\text{des}} = 1.35$) and stall ($q_{m,\text{ref}}/q_{m,\text{des}} = 0.55$).

It is evident that the axial velocity distributions vary at different circumferential locations at the off-design conditions. This leads to the assumption that also the flow angle distributions differ at different circumferential locations leading to an uneven mass distribution at different circumferential locations. This then leads to a more distorted volute flow increasing losses there.

Further research is needed to study the optimum pinch i.e. large enough to suppress the loss mechanisms as thoroughly as possible, without separating the flow in the diffuser. In addition, more general outlines about the width increment are needed to give the design engineers better tools to estimate the sufficient width reduction. Also, the effect of other parameters affecting the diffuser-impeller interaction at off-design conditions must be studied further. This is especially relevant if a compressor stage with wide operating range is desired.

References

- ASME PTC 10 (1965). *ASME Power Test codes, compressors and exhausters*. The American Society of Mechanical Engineering.
- Backman, J., Reunanen, A., Saari, J., Turunen-Saaresti, T., Sallinen, P., and Esa, H. (2007). Effects of Impeller Tip Clearance on Centrifugal Compressor Efficiency. In *Proceedings of ASME Turbo Expo, GT2007-28200, May 14-17, Montreal, Canada*.
- Cellai, A., Ferrara, G., Ferrari, L., Mengoni, C., and Baldassarre, L. (2003). Experimental Investigation and Characterization of the Rotating Stall in a High Pressure Centrifugal Compressor Part III: Influence of Diffuser Geometry on Stall Inception and Performance (2nd Impeller Tested). In *Proceedings of ASME Turbo Expo, GT2003-38390, June 16-19, Atlanta, USA*.
- Denton, J. (1993). Loss mechanisms in turbomachines. *Journal of Turbomachinery*, 115:621–656.
- Di Liberti, J.-L., Wilmsen, B., and Engeda, A. (1996). The effect of the vaneless diffuser width on the performance of a centrifugal compressor. FED-vol. 237, pages 797–803. Fluids Engineering Division Conference, ASME.
- DIN 1952 (1971). *Durchflußmessung mit genormten Düsen, Blenden und Venturidüsen*. Deutsches Institut für Normung e.V.
- Eisenlohr, G. and Chladek, H. (1994). Thermal Tip Clearance Control for Centrifugal Compressor of an APU Engine. *Journal of Turbomachinery*, 116:629–634.
- Ferrara, G., Ferrari, L., Mengoni, C. P., Lucia, M. D., and Baldassarre, L. (2002a). Experimental investigation and characterization of the rotating stall in a high pressure centrifugal compressor: Part I: Influence of diffuser geometry on stall inception. In *Proceedings of ASME Turbo Expo, GT2002-30389, June 3-6, Amsterdam, The Netherlands*.

- Ferrara, G., Ferrari, L., Mengoni, C. P., Lucia, M. D., and Baldassarre, L. (2002b). Experimental investigation and characterization of the rotating stall in a high pressure centrifugal compressor: Part II: Influence of diffuser geometry on stage performance. In *Proceedings of ASME Turbo Expo, GT2002-30390, June 3-6, Amsterdam, The Netherlands*.
- ISO 5389 (1992). *Turbocompressors - Performance test code*. International Standardization Organization.
- Issac, J. M., Sitaram, N., and Govardhan, M. (2003). Performance and wall static pressure measurements on centrifugal compressor diffusers. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, 217:547–558.
- Jaatinen, A., Grönman, A., and Turunen-Saaresti, T. (2012a). Effect of diffuser width and tip clearance on the static pressure distribution in a vaneless diffuser of a high-speed centrifugal compressor. In *Proceedings of the 10th International Conference on Turbochargers and Turbocharging, May 15-16, London, UK*.
- Jaatinen, A., Grönman, A., Turunen-Saaresti, T., and Røyttä, P. (2011). Effect of Vaneless Diffuser Width on the Overall Performance of a Centrifugal Compressor. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, 225:665–673.
- Jaatinen, A., Turunen-Saaresti, T., Grönman, A., Røyttä, P., and Backman, J. (2012b). Experimental study of the effect of the tip clearance to the diffuser flow field and stage performance of a centrifugal compressor. In *Proceedings of ASME Turbo Expo, GT2012-68445, June 11-15, Copenhagen, Denmark*.
- Kim, Y., Engeda, A., Aungier, R., and Amineni, N. (2002). A centrifugal compressor stage with wide flow range vaned diffusers and different inlet configurations. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, 216:307–320.
- Ludtke, K. (1983). Aerodynamic tests on centrifugal process compressors - the influence of the vaneless diffuser shape. *Journal of Engineering for Power*, 105:902–909.
- Mashimo, T., Watanabe, I., and Ariga, I. (1979). Effects of fluid leakage on performance of a centrifugal compressor. *Journal of Engineering for Power*, 101:337–342.
- Palmer, D. L. and Waterman, W. F. (1995). Design and development of an advanced two-stage centrifugal compressor. *Journal of Turbomachinery*, 117(2):205–212.
- Pampreen, R. (1973). Small turbomachinery compressor and fan aerodynamics. *Journal of Engineering for Power*, pages 251–256.
- Pinarbasi, A. (2009). Turbulence measurements in the inlet plane of a centrifugal compressor vaneless diffuser. *International Journal of Heat and Fluid Flow*, 30:266–275.
- Pinarbasi, A. and Johnson, M. W. (1994). Detailed Flow Measurements in a Centrifugal Compressor Vaneless Diffuser. *Journal of Turbomachinery*, 116:453–461.
- Pinarbasi, A. and Johnson, M. W. (1995). Off-Design Flow Measurements in a Centrifugal Compressor Vaneless Diffuser. *Journal of Turbomachinery*, 117:602–608.
- Pinarbasi, A. and Johnson, M. W. (1996). Detailed Stress Tensor Measurements in a Centrifugal Compressor Vaneless Diffuser. *Journal of Turbomachinery*, 118:394–399.

- Schleer, M. and Abhari, R. S. (2008). Clearance effects on the evolution of the flow in the vaneless diffuser of a centrifugal compressor at part load condition. *Journal of Turbomachinery*, 130:9.
- Senoo, Y. and Ishida, M. (1987). Deterioration of compressor performance due to tip clearance of centrifugal impellers. *Journal of Turbomachinery*, 109:55–61.
- Shaaban, S. and Seume, J. (2007). Aerodynamic performance of small turbocharger compressor. In *Proceedings of ASME Turbo Expo, GT2007-27558, May 14-17, Montreal, Canada*.
- VDI 2045 Part 1 (1993). *Abnahme- und Leistungsversuche an Verdichtern, Versuchsdurchführung und Garantievergleich (Acceptance and Performance Test on Turbo Compressors and Displacement Compressors, Test Procedure and Comparison with Guaranteed Values)*. Verein Deutscher Ingenieure.
- VDI 2045 Part 2 (1993). *Abnahme- und Leistungsversuche an Verdichtern, Grundlagen und Beispiele (Acceptance and Performance Test on Turbo Compressors and Displacement Compressors, Theory and examples)*. Verein Deutscher Ingenieure.
- You, M., Wang, M., Moin, P., and Mittal, R. (2007). Large-eddy simulation analysis of mechanisms for viscous losses in a turbomachinery tip-clearance flow. *Journal of Fluid Mechanics*, 586:177–204.