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THE EFFECTS OF REYNOLDS NUMBER AND ROUGHNESS ON COMPRESSOR PERFORMANCE

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ABSTRACT

An empirical correlation (due to Casey and Robinson (2011)) for the effect of Reynolds number and roughness on the performance of compressors is examined and refined. The original correlation quantifies the inefficiency due to friction losses with an empirical coefficient B_{ref} which differs for different machine types. The variation of this coefficient was originally given as a function of the specific speed and determined from extensive test data for many compressors, in which the Reynolds number, roughness and size were varied. In this paper the value of the empirical coefficient is determined from a theoretical analysis and the experimental data is reanalysed and extended to provide an improved correlation.

Two theoretical approaches are given. The first determines the inefficiency using a method due to Traupel to give the frictional dissipation losses as a function of flow coefficient for a range of radial compressor stages. The second uses a well-known correlation for global efficiency, which is corrected for the non-frictional losses to derive the inefficiency due to the friction losses alone. The original test data and some additional data has also been analysed with a slight modification of the equation for the friction factor as a function of Reynolds number and roughness, and this removes some of the scatter in the experimental data.

On this basis three independent methods for determining the relevant coefficient are now available. A suggestion is made for an improved correlation based on these three approaches.

NOMENCLATURE

A,B	Coefficients in correction equations (-)
A	Also surface area of wetted walls (m²)
b	Channel width or height (m)
c	Chord length (m), also Absolute velocity (m/s)
c_L, c_D	Lift and drag coefficients (-)
D_2	Tip diameter (m)
f	Friction factor (-)
h	Specific enthalpy (J/kg)
j	Dissipation loss (-)
k_s	Sand roughness (m)
l	Flow path length (m)
m	Meridional direction (-)
$\dot{m}$	Mass flow (kg/s)
n	Blade number (-)
P	Perimeter (m), also Blending function (-)
r	Radius (m)
Ra	Centre-line average roughness (m)
ReC	Chord Reynolds number (-)
s	Specific entropy (J/(kg K))
S	Entropy (J/K)
T	Temperature (K)
u	Blade speed (m/s)
V	Volume flow rate (m³/s)
w	Relative velocity (m/s)
η	Efficiency (-)
ϕ	Flow coefficient (-)
ρ	Density (kg/m³)
λ	Work coefficient (-)

Subscripts

1	condition 1, or inlet of impeller
2	condition 2, or outlet of impeller
a	Reynolds independent loss fraction (-)
b	Fraction of Reynolds dependent losses (-)
p	polytropic
ref	at reference conditions
t	total
m	mean

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INTRODUCTION

The changes in compressor performance due to changes in Reynolds number or roughness involve a change in efficiency, a shift in flow and a change in the pressure rise. Although these changes are small enough to be neglected in many cases, there are many turbomachinery applications where the changes are large. Casey (1985) includes test data of a low specific speed radial compressor where the efficiency increases by more than 15 % points due to a ten-fold change in inlet pressure. Similar large changes in efficiency are quoted for an axial fan with changes in rotational speed at low speeds by Hess and Pelz (2010).

Three different issues can arise due to this change in performance with Reynolds number:

- The size effect. Can we predict the accurate performance of a scaled machine from test data on a geometrically similar machine? This issue occurs when model tests on small scale rigs are used as a basis to determine the performance of a full-scale machine, but may also be relevant when scaling down from larger to smaller machines where less experience is available, see Casey et al. (2013).
- The Reynolds effect. What changes in performance can be expected if we change the flow conditions (density, viscosity, speed) relative to those used in a test rig? This issue includes altitude effects in aero-engines, a change in the operating gas or the operational pressure levels for a compressor tested on atmospheric air, and viscosity effects at different temperatures in pumps. On low-speed machines with low Mach numbers a change in performance due to speed variations may also be attributed to a Reynolds effect.
- The roughness effect. What is the change in performance due to changes in the absolute surface roughness? Does the associated performance improvement make it worthwhile to improve the surface finish of the wetted flow surfaces of a machine? At what roughness level is no further change in performance expected? What degradation in performance can be expected due to surface erosion from particles in the flow?

In this paper these issues are examined by developing some improvements to the empirical correlation (due to Casey and Robinson (2011)) for the effect of Reynolds number, roughness and size on the performance of compressors.

The paper is organised as follows. The first section describes the original correlation. The form of the correction equation is discussed and its special features are highlighted. An analytical section then describes two theoretical approaches to determine the value of the empirical coefficient. The first approach determines the inefficiency due to the friction losses as a function of flow coefficient by carrying out a loss analysis for a range of radial compressor stages. The second is based on well-known correlations for global efficiency, which are then corrected for the effect of non-frictional losses to derive the inefficiency due to the friction losses alone. A subsequent section considers the test data used for the original correlation with additional data from other sources. This has also been reanalysed with an improved equation for the friction factor as a function of Reynolds number and roughness, and this removes some of the scatter in the experimental data. On this basis, there are now three independent methods for determining the relevant coefficient in the correlation equation available. A suggestion is made for an improved correlation based on these three approaches.

THE CORRECTION PROCEDURE

Many issues related to developing a unified correction method suitable for all types of compressors are discussed by Casey and Robinson (2011), where a general correction equation was derived suitable for examining changes in performance due to changes in Reynolds number or roughness of the individual components of a multistage or single stage axial or radial machine. Here the basic equation is derived in a much simpler form to that of the original paper to draw out the significance of the terms in preparation for the work which follows. If we consider the dissipation losses in the stage an equation for the inefficiency of the stage can be derived as

$$1-\eta_p = \frac{j}{\Delta h_t} \quad with \quad j = Tds. \tag{1}$$

In this equation, j is the total dissipation loss and Δh_t is the total enthalpy change over the stage. For simplicity we argue that the losses can be split into two different groups. There are losses which remain constant when the Reynolds number is changed and there are losses related to frictional effects, which can be expected to change with the Reynolds number. The first group includes shock losses, mixing losses, leaving losses, secondary kinetic energy losses and clearance loss and is given the suffix a. The second group includes frictional dissipation on the wetted surfaces of the stage components (suffix b), so that we obtain

$$1-\eta_p = \frac{j_a}{\Delta h_t} + \frac{j_b}{\Delta h_t} = A + B. \tag{2}$$

In this equation the first term, A, represents the contribution to the inefficiency due to those loss sources that are independent of the Reynolds number and the term B represents the inefficiency due to frictional losses that are Reynolds dependent, being mainly the profile losses and endwall friction losses, but may also include disc friction losses where these are significant.

In many correction methods for Reynolds number a problem arises due to the fact that the efficiency changes with the Reynolds number. To avoid this problem in the scaling equations a reference condition needs to be defined. If this is not done then a correction of the efficiency from the model to the full-scale does not necessarily give the same correction as the change from full-scale to the model, see the discussion in Simon and Bülskämper (1984). The reference condition used here is the same as that given by Casey and Robinson (2011) at a defined high Reynolds number with a hydraulically smooth surface and we may write that at this reference condition

$$1-\eta_{p,ref} = A + B_{ref}\,. \tag{3}$$

We now assume that the change in the Reynolds dependent losses with the Reynolds number can be scaled with the change in friction factor of a representative flow

$$B = B_{ref}\frac{f}{f_{ref}}\,. \tag{4}$$

On this basis the effect on the inefficiency of a change in the friction factor due to a change in the Reynolds number or relative roughness can be calculated for a typical stage as

$$\Delta\eta_p = -B_{ref}\frac{\Delta f}{f_{ref}} = -(1-\eta_{p,ref} - A)\frac{\Delta f}{f_{ref}}\,. \tag{5}$$

Note that the coefficient B_{ref} is simply the inefficiency due to friction losses at the reference conditions. This is the inefficiency ($1-\eta_{p,ref}$) at the reference conditions reduced by the losses that are not dependent on the Reynolds number (A). Scaling with the whole inefficiency (by assuming $A = 0$) neglects to take into account that some losses are not dependent on Reynolds number. Scaling with a measured efficiency on a particular characteristic leads to slightly different scaling factors for each characteristic.

A key feature of this equation is that it leads to an approach to determine the coefficients from test data. If the test data for inefficiency is plotted as a function of the representative friction factor, a straight line should result; the slope of the line determines B (actually B_{ref}/f_{ref}) and the intersection with the axis determines A. A good example is given in Figure 1, which is a test case of an axial fan taken from Hess (2010). The Reynolds number has been varied by testing at different speeds and sizes, but this case is particularly interesting as the high quality measurements include test data at different clearance levels. The linear change of η_p with friction factor can be identified and in this case a value $B_{ref} = 0.05$ is suggested by each clearance data set, but different values of the Reynolds independent losses (A), occur for the different clearances, as would be expected. The parameter A varies between $A = 0.06$ to 0.16 depending on the clearance. Note that the variation of efficiency in this case is precisely as expected from equation 3, in that the change in clearance does not change the value of B_{ref} determined in this procedure which demonstrates the validity of the approach.

A second contrasting example is shown in Figure 2 from the test data of Kawakubo et al. (2008) in which the roughness of a radial compressor was changed. This has been done for the impeller, diffuser and the inlet casing by sand blasting. In this case the higher roughness levels do not cause a change in the value of A, as expected, but shift the friction factor for a given Reynolds number to higher values along the same line.

A similar procedure was used to calibrate the value of the coefficients against flow coefficient for over 31 different compressors to determine the empirical correlation for B_{ref} given by Casey and Robinson (2011).

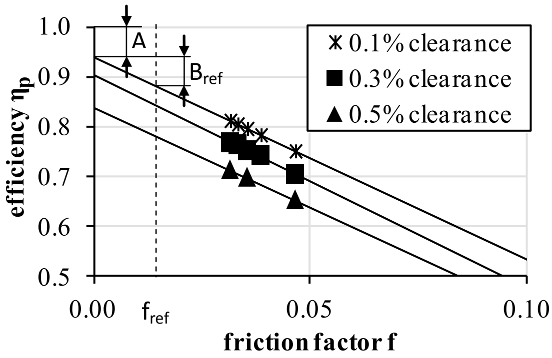

Figure 1: Variation of efficiency with friction factor for an axial fan tested with different speed, sizes and clearance with data from Hess (2010).

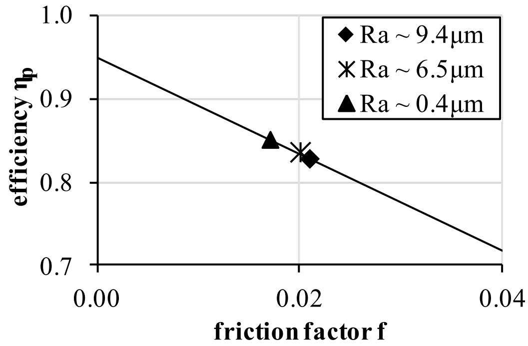

Figure 2: Variation of efficiency with friction factor for a radial compressor tested at different roughness levels with data from Kawakubo et al. (2008).

DERIVATION OF THE COEFFICIENTS IN THE CORRECTION EQUATION

In this section three different approaches are given to derive the value of the coefficient B_{ref} in equation (5). The first approach attempts to determine the value of B directly, by examining the frictional dissipation losses as a function of flow coefficient using an analytical method suggested by Traupel (1966). This determines an approximation for the inefficiency due to this source for a range of radial compressor stages. The second approach determines B indirectly by determining the total inefficiency ($1-\eta_{p,ref}$) from a well-known correlation for global efficiency, which is then corrected with an estimate for the non-frictional losses (A) giving $1-\eta_{p,ref} - A$. The third approach is a reexamination of the original test data with additional data added, in which the data has been reanalysed with a slight modification to the way in which the friction factor is calculated.

Determination of B from loss analysis

A loss analysis based on an approach by Traupel (1966) suggests that the total dissipation loss for a small section of a flow channel may be estimated as

$$TdS = (\rho / 2)c_d \int w^3 dA \tag{6}$$

where c_d is a mean dissipation coefficient (and is related to the skin friction factor, see Traupel equation 3.15(8)), w is the local relative velocity and A is the surface area of the wetted walls (not the flow area!), which is the perimeter (P) times the flow path length (dl)

$$dA = Pdl. \tag{7}$$

If we consider a 2D radial impeller as in Figure 3 we can derive a simple formula for this. For one blade passage at radius r follows

$$P = 2\left(\frac{2\pi r\cos(\beta)}{n} + b\right) \tag{8}$$

where n is the number of blades and b is the width of the flow channel. The factor two outside of the bracket is related to the two blade surfaces and the two endwall surfaces. This formulation is taken to be approximately valid for 3D impellers if we consider the radius as the mean radius. This can also be expressed as

$$P = 2b\left(1 + \frac{2\pi r_m \cos(\beta)}{bn}\right) \tag{9}$$

and the second term in the bracket is effectively the ratio of the surface area of the end-walls to the area of the profile.

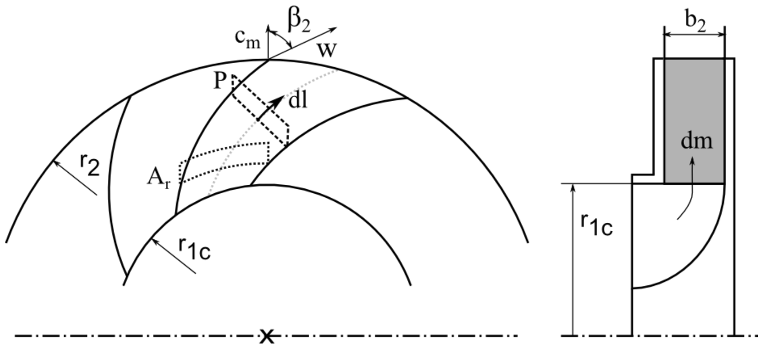

Figure 3: Simple 2D radial impeller model

By using equations (6), (7) and (9) we derive that

$$TdS = (\rho/2)c_d \int w^3 2b\left(1 + \frac{2\pi r \cos(\beta)}{bn}\right)dl. \tag{10}$$

This can be expressed in terms of global dimensionless parameters as

$$TdS = (\rho/2)c_d u_2^3 D_2 \int \left(\frac{w}{u_2}\right)^3 2\left(\frac{b}{D_2}\right)\left(1 + \frac{2\pi r \cos(\beta)}{bn}\right)dl. \tag{11}$$

Equation (11) needs to be made specific by dividing by the mass flow. In our case the local mass flow is given by

$$\dot{m} = (\rho c_m A_r)/n \tag{12}$$

where A_r represents the area of the flow channel normal to the meridional direction

$$A_r = 2\pi r_m b. \tag{13}$$

With this we arrive at the final formulation for the specific entropy loss as

$$\begin{aligned} Tds &= TdS/\dot{m} = (\rho/2)c_d u_2^3 D_2 n/(\rho c_m 2\pi r_m b)\int \left(\frac{w}{u_2}\right)^3 2\left(\frac{b}{D_2}\right)\left(1 + \frac{2\pi r_m \cos(\beta)}{bn}\right)dl \\ Tds &= \left(\frac{n c_d u_2^3}{2\pi}\right)\int \left(\frac{w}{u_2}\right)^3 \left(\frac{u_2}{c_m}\right)\left(\frac{D_2}{r_m}\right)\left(1 + \frac{2\pi r_m \cos(\beta)}{bn}\right)\left(d\frac{l}{D_2}\right). \end{aligned} \tag{14}$$

Other formulations might be more convenient to work with and if we note that

$$\begin{aligned} dl &= dm/\cos\beta \\ w &= c_m/\cos\beta \end{aligned} \tag{15}$$

we get a final expression as

$$Tds = \left(\frac{nc_d u_2^2}{2\pi}\right)\int\left(\frac{c_m}{u_2}\right)^2\left(\frac{1}{\cos\beta}\right)^4\left(\frac{D_2}{r_m}\right)\left(1+\frac{2\pi r_m\cos(\beta)}{bn}\right)\left(d\frac{m}{D_2}\right). \tag{16}$$

In order to calculate B_{ref} we need to introduce the definition of the work coefficient which can be expressed as

$$\Delta h_t = h_{t,2} - h_{t,1} = \lambda u_2^2. \tag{17}$$

Final expression

By combining equations (16), (17) and comparing with equation (2), the final expression for the Reynolds dependent efficiency loss is given as

$$\frac{j_b}{\Delta h_t} = B = \left(\frac{nc_d}{2\pi\lambda}\right)\int\left(\frac{c_m}{u_2}\right)^2\left(\frac{1}{\cos\beta}\right)^4\left(\frac{D_2}{r_m}\right)\left(1+\frac{2\pi r_m\cos(\beta)}{bn}\right)\left(d\frac{m}{D_2}\right). \tag{18}$$

Note that the term related to the area of the endwalls relative to the blade profile appears as expected in this formulation. Note also that if we replace the dissipation coefficient directly by the friction factor for a representative flow at reference conditions we get

$$B_{ref} = \left(\frac{nf_{ref}}{2\pi\lambda}\right)\int\left(\frac{c_m}{u_2}\right)^2\left(\frac{1}{\cos\beta}\right)^4\left(\frac{D_2}{r_m}\right)\left(1+\frac{2\pi r_m\cos(\beta)}{bn}\right)\left(d\frac{m}{D_2}\right). \tag{19}$$

This equation could be applied accurately to the geometry of any stage to determine the value of B_{ref}. Unfortunately the data being used here is extracted from the literature and no geometrical details are provided to be able to do this. So in order to carry out the integration we need to make certain assumptions about the variation of the parameters within the integration and how they vary through the length of a typical impeller. For a 2D impeller in incompressible flow as shown in Figure 3 the following assumptions are made for solving the integral of equation (19):

- The upper integration limit is defined by 0.5 because the impeller ends at $r_2/D_2 = 0.5$.
- The lower integration limit is defined by a correlation for $r_{1c}/D_2 = f(\phi)$ based on an analysis of impellers in various publications (Aungier (2000), Bygrave et al. (2010)).
- A linear variation of the blade angle β between r_{1c} and r_2 is assumed.
- The value of the angle β at position r_{1c} and r_2 are functions of the flow coefficient ϕ. Thereby it is assumed that the inlet angle $\beta_1 = 60°$ for flow coefficients higher than $\phi = 0.047$ and it increases linearly to $\beta_1 = 70°$ for a flow coefficient of $\phi = 0.016$.
- The same is done for the outlet angle β_2 with the limits given in Lüdtke (2004). Thereby three different reference points are specified: $\beta_2 = 40°$ for $\phi >= 0.047$, $\beta_2 = 50°$ for $\phi = 0.016$, $\beta_2 = 70°$ for $\phi = 0.01$. A linear variation in between is assumed.
- For the blade number the correlation of Pfleiderer and Petermann (1991) is used.
- For the ratio of the relative velocity w at the inlet to the blade tip speed u_2 a typical value of $w_1/u_2 = 0.6$ for radial compressors is used, the relative velocity at the outlet is calculated using a DeHaller number of 0.6. Throughout the channel a linear variation of w/u between the integration limits given above is assumed.
- As a mean value a work coefficient $\lambda = 0.6$ is used in this calculation.
- In the case of a purely radial geometry as shown in Figure 3 the term $d(m/D_2)$ simplifies to $d(r/D_2)$ and from D_2/r_m we get D_2/r.

Performing the integration with the assumptions given above provides an analytical derivation for the variation of the coefficient B_{ref} as a function of the flow coefficient ϕ. The results can be seen in Figure 4, where they are compared to the new analysis of the test data and with a correlation based on this, see below. The agreement is excellent considering the rather global assumptions made.

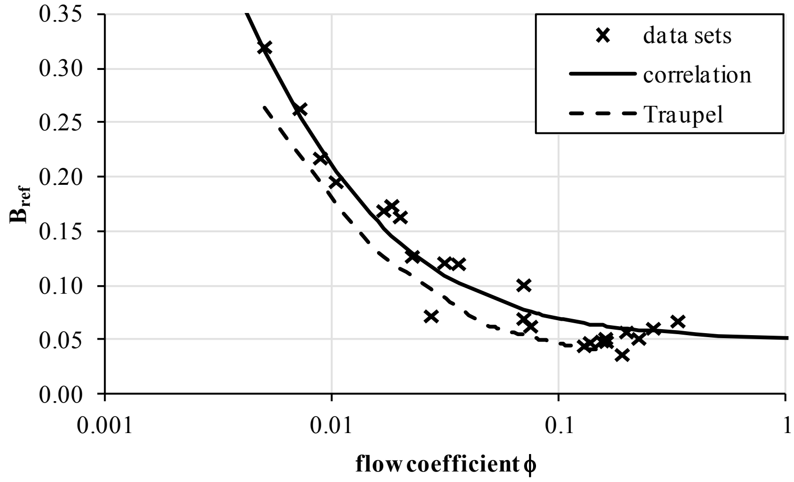

Figure 4: Variation of B_{ref} calculated from simple Traupel loss approach

Determination of B from efficiency correlations

The second approach for the determination of B is to make use of efficiency correlations which provide the expected efficiency of the stage as a function of specific speed or flow coefficient, for example the correlation of Bommes et al. (2003). The paper of Casey et al. (2010) gives the full equations of this correlation, and demonstrates that it has a wider application into the mixed flow region than other similar correlations. If we assume that we have a simple correlation for the overall efficiency of well-designed stages as a function of specific speed or flow coefficient, then these can be used together with equation (3) to define the value of B_{ref} as follows

$$B_{ref} = (1 - \eta_{p,ref} - A). \tag{20}$$

Unfortunately such correlations do not specifically define the fraction of losses that are independent of Reynolds number (A), so we need to estimate this. Typically such correlations are derived for well designed machines with the smallest possible value of A (that is minimum possible leakage losses and leaving losses), and for large machines that are hydraulically smooth, so that the value of B may be close to that of B_{ref}. If we assume that the Reynolds independent loss fraction is 35% of the total inefficiency, then we obtain the following curves for B_{ref} using the Bommes correlation of efficiency and the trend of this curve also agrees very well with the trend of the experimental data. Other efficiency correlations have been examined and also produce similar trends and because of this are not included here.

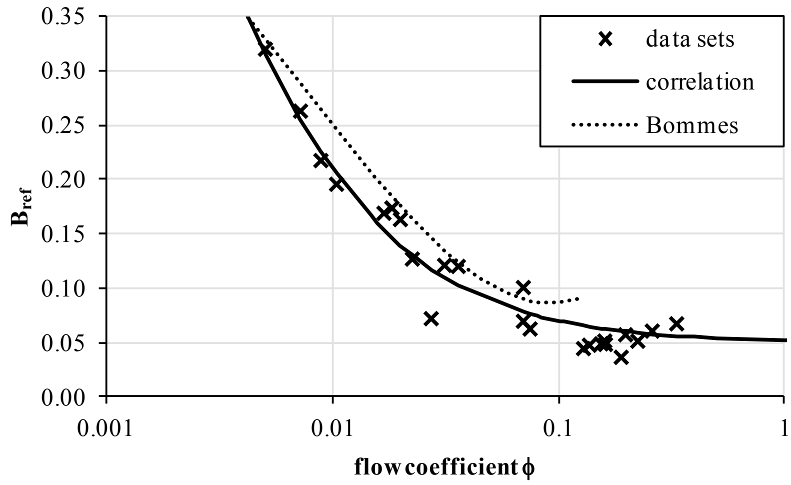

Figure 5: Variation of B_{ref} calculated from the Bommes et al. (2003) correlation for radial and mixed flow stages with the assumption, that one third of the losses is Reynolds independent.

Determination of B from test data

The original test data of Casey and Robinson (2011) was reanalyzed and the calculation procedure was refined. Three particular changes were made. Firstly, some datasets with uncertainties were excluded from the new correlation, in particular those cases where some of the crucial geometrical information was not available or where the determination of the slope as shown in Figure 1 was uncertain or inconsistent in itself. Uncertainty arises if there are only a small number of test points and if they are closely spaced in terms of the equivalent friction factor. Figure 1 shows data which appears excellent for the purpose. There are a sufficient number of test points to allow the value of B_{ref} to be determined with little uncertainty, and the data is internally consistent, as the effect of clearance on the parameter (A) is precisely as one would expect. Note that many of the cases now excluded actually agree well with the correlation but do not satisfy this criterion. Secondly new data sets have been added in order to improve the determination of the correlation coefficients, Hess (2010), Kawakubo et al. (2008) and Gerke and Gikadi (2006). The whole data-set collection now consists of 31 cases. The third change is that the value of the sand roughness used in the calculations has been modified. For machined surfaces the sand roughness has been calculated from the value of the centre line average roughness using $k_s = 1*Ra$. Previously a factor of 5 was used in this equation. This change has been made based on a reanalysis of the test data in which this factor was varied. In addition the equations for the friction factor used have been traced back to a diagram and an equation in the paper by Granville (1958) where it seems clear that the roughness values are the actual engineering roughness. In cases where the surfaces are not machined but are painted or treated in some way a different rule is needed for the roughness. Figure 6 shows the variation of the coefficient B_{ref} with the flow coefficient and an equation for the friction factor and for the correlation is given in the appendix. Two lines indicate a ±25% offset margin of the correlation. It can be seen that the major part of the data points lay within the offset margin.

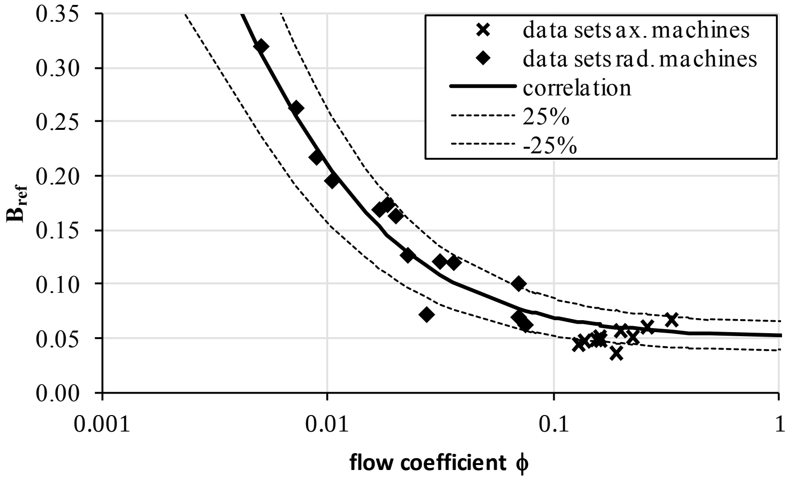

Figure 6: Variation of B_{ref} calculated from measurement data

DICUSSION

As can be seen in Figures 4, 5 and 6, the trends of the factor B_{ref} determined by the different approaches agree very well with each other. The variation of B_{ref} with flow coefficient and the slope of the variation are similar in each of the approaches considered.

If the correlation taken from the measurement data is taken as a reference then the analytical solution slightly underestimates the parameter B_{ref}. This may be because this approach examines the impeller only (where most of the losses occur because of the high flow velocities) and does not consider the presence of a diffuser or return channel. For ideal flow coefficients (around 0.09) the assumptions in the integration process may also need to be modified because the leading edge and thereby the lower integration limit moves further upstream. Simple modification of the integration shows that this effect can be considered, and shows the right trend, but no results are shown because this is not within the scope of the work presented above.

The method is still under development to remove further weaknesses related to the accuracy when it is used to estimate the effect of roughness changes. Figure 7 shows the location of all the measurement points in the friction factor equations. Crosses are given for most of the test points, and as can be seen nearly all of these cases lie on the curve for a hydraulically smooth surface. This is to be expected as the stage efficiency would be lower if the roughness was higher and in such a case the manufacturer would certainly improve the machining to provide a smoother surface. Four cases are highlighted specifically; these are the cases with a variation in roughness, and the pump test case. In these cases the stages are not hydraulically smooth. The data for these cases agrees well with the correlation with the coefficients currently used, but additional work is proceeding to assess improved methods of taking the roughness into account. More data is needed at high Reynolds number with high roughness to test the new equations and so these are not published in this paper.

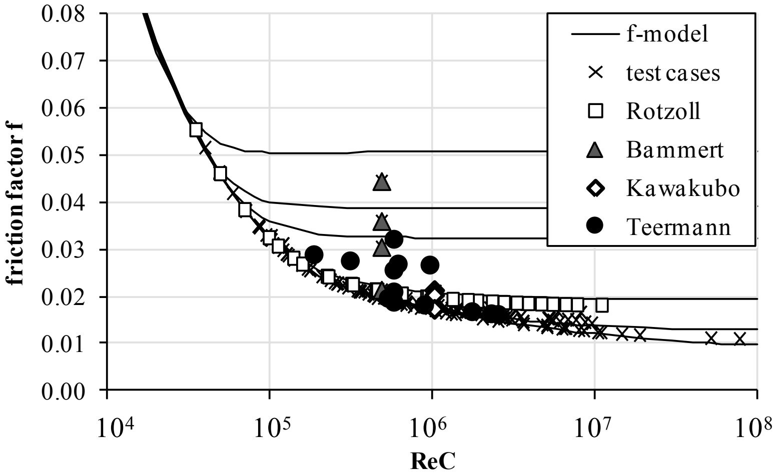

Figure 7: Variation of friction factor for all test cases

Many questions besides the consideration of roughness are still open. For determining the influence of the impeller type (shrouded or unshrouded), the impact of the diffuser (vaned or vaneless), the volute or return channel or disk friction and leakage flows more research is needed. This may improve this general procedure by subdividing B_{ref} into the different $B_{ref,i}$ for each component. Access to highly accurate and detailed measurement data would be needed for this.

CONCLUSIONS

The empirical correlation suggested by Casey and Robinson (2011) has been improved and extended. Three independent ways for calculating the variation of the factor B_{ref} towards small flow coefficients are presented and show good agreement with each other.

An analytical loss analysis procedure suggested by Traupel has confirmed the tendency of a strong increase of the parameter B_{ref} towards smaller flow coefficients. Low flow coefficient stages are more sensitive to the effect of Reynolds number as the frictional losses increase due to an increase in the ratio of the endwall surface area to that of the blades as the flow coefficient decreases. Together with the formulation of the limit of B_{ref} towards high flow coefficients, already given by Casey and Robinson (2011), two analytical procedures are now available to explain the asymptotic behaviour of B_{ref} for high and low flow coefficients. The approach given could be adapted to calculate the value of B_{ref} for a particular stage, without the need for a correlation, if geometry data would be available.

An approach using efficiency correlations to determine the value of B_{ref} is also shown to be consistent with the other methods. The measurement data was refined and additional measurement data taken from the open literature also confirms the trend of B_{ref}, and a new correlation based solely on the flow coefficient is given.

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REFERENCES

Aungier, R. H. (2000), Centrifugal Compressors: A Strategy for Aerodynamic Design and Analysis, ASME Press.

Bommes, L., Fricke, J. and Grundmann, R. (2003), Ventilatoren, Vulkan –Verlag Essen, Germany.

Bygrave, J., Villanueva, A., Enos, E., Saladino, A., Serrino, D. and Prosser, W. (2010), 'Upgrading the performance of a centrifugal barrel compressor family', ASME Paper GT2010-23767. Proceedings of the ASME Turbo Expo 2010, June 14-18, 2010, Glasgow, Scotland.

Casey, M. V. (1985), 'The effect of Reynolds number on the efficiency of centrifugal compressor stages', Trans. ASME, J. Turbomachinery 107, 541–548.

Casey, M. V., Krähenbuhl, D. and Zwyssig, C. (2013), The design of ultrahigh speed miniature centrifugal compressors, in F. Martelli, F. Heitmeir and M. Manna, eds, 'Proc. 10th European. Conference on Turbomachinery, Fluid Dynamics and Thermodynamics', Lappeenranta, Finland, April 2013.

Casey, M. V. and Robinson, C. J. (2011), 'A unified correction method for Reynolds number, size, and roughness effects on the performance of compressors', Proc. IMechE, Part A: J. Power and Energy 225(7), 864–876.

Casey, M., Zwyssig, C. and Robinson, C. (2010), 'The Cordier line for mixed flow compressors', ASME Paper GT2010-22549. Proceedings of the ASME Turbo Expo 2010, June 14-18, 2010, Glasgow, Scotland, UK.

Gerke, S. and Gikadi, T. (2006), Untersuchungen zur Wirkungsgradaufwertung in Abhängigkeit von der Oberflächenrauhigkeit und der Spaltweite der Ventilatoreinlaufdüse, Technical report, unpublished, FLT Nr. 210.

Gülich, J. F. (2003), 'Effect of Reynolds number and surface roughness on the efficiency of centrifugal pumps', Trans. ASME, J. Fluids Eng. 125(4), 670–679.

Granville, P. (1958), 'The frictional resistance and turbulent boundary layer of rough surfaces', Journal of Ship Research pp. 52–74.

Hess, M. (2010), Aufwertung bei Axialventilatoren - Einfluss von Reynolds-Zahl, Rauheit, Spalt und Betriebspunkt auf Wirkungsgrad und Druckziffer, PhD thesis, Technische Universität Darmstadt.

Hess, M. and Pelz, P. F. (2010), On reliable performance prediction of axial turbomachines, GT2010-22290, Proceedings of ASME Turbo Expo 2010, June 14-18, 2010, Glasgow, Scotland.

Kawakubo, T., Numakura, R. and Majima, K. (2008), Prediction of surface roughness effects on centrifugal compressor performance, in 'Proceedings of FEDSM2008 ASME Fluids Engineering Conference', American Society of Mech. Eng.

Lüdtke, K. H. (2004), Process centrifugal compressors: Basics, functions, operation, design, application, Springer, Berlin.

Pfleiderer, C. and Petermann, H. (1991), Strömungsmaschinen, 6 edn, Springer, Berlin.

Simon, H. and Bülskämper, A. (1984), 'On the evaluation of Reynolds number and relative surface roughness effects on centrifugal compressor performance based on systematic experimental investigations', Trans. ASME, J. Eng. for Gas Turbines and Power 106(2), 489–498.

Traupel, W. (1966), Thermische Turbomaschinen, 2 edn., Springer, Berlin.

APPENDIX:

Following Gülich (2003), the equations for the skin friction can be written as

$$c_{f,lam} = \frac{k_1}{Re^{0.5}}, \quad c_{f,turb} = \frac{0.136}{\left\{-\log_{10}\left(0.2\frac{k_s}{c} + \frac{12.5}{Re}\right)\right\}^{2.15}}, \quad Re = \frac{w_1 c}{\nu}. \tag{21}$$

These are combined by using a blending function P and we get

$$f = 4c_f, \quad c_f = Pc_{f,lam} + (1-P)c_{f,turb}, \quad P = \frac{1}{1+e^{-t}}, \quad t = k_2\left(\frac{c_{f,lam}}{c_{f,turb}} - 1\right). \tag{22}$$

P varies between 0 for turbulent and 1 for laminar flow and blends the two regions together. A value of 2.656 for k_1 and 5 for k_2 are used. The recent procedure of Casey and Robinson (2011) is thereby slightly modified by using

$$k_s = 1Ra. \tag{23}$$

The following equations determine the new correlation for B_{ref} which are given as

$$\phi = \frac{\dot{V}}{u_2 D_2^2}, \quad B_{ref} = 0.05 + \frac{0.002}{\phi + 0.0025}. \tag{24}$$